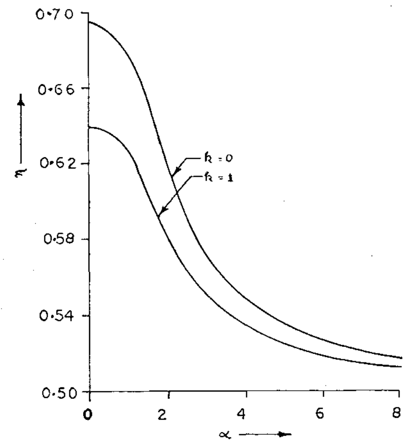

Fig. 1 Optimum power law exponent.

Now in order that the ballistic factor be minimum, $dI/dn = 0$, i.e.,

$$\frac{24n^4 - 8n^3 - 6n^2 + k(12n^4 - 12n^3 + n^2 + 2n)}{4(2n-1)^2} + \frac{\alpha^3}{(n+1)^2} = 0 \quad (8)$$

This gives the values of n for given values of the factor α. Knowing the value of n we can calculate the missile shapes of minimum ballistic factor and also from Eq. (7) the corresponding values of I.

Now Eq. (8) is a general expression and we can deduce all the previous results from here. The following four cases are possible. Case 1: When $k = 0$, $\alpha = 0$ (Newton law neglecting friction); $n = 0.694$ as also obtained by Fink.[2] Case 2: When $k = 1$, $\alpha = 0$ (Newton-Busemann law neglecting friction); $n = 0.637$ as also obtained by Fink.[2] Case 3: When $k = 0$, $\alpha \neq 0$ (Newton law accounting for friction); then the optimum value of n is given by

$$\frac{12n^4 - 4n^3 - 3n^2}{2(2n-1)^2} + \frac{\alpha^3}{(n+1)^2} = 0 \quad (9)$$

a result also obtained by Miele and Huang.[3] Case 4: When $k = 1$ and $\alpha \neq 0$ (Newton-Busemann law accounting for friction) is not reported so far and is being considered here. The results obtained are compared with those given by Case 3 due to Miele and Huang. In this case the relation between n and α is given by

$$\alpha = \left[\frac{(-36n^4 + 20n^3 + 5n^2 - 2n)(n+1)^2}{4(2n-1)^2}\right]^{1/3} \quad (10)$$

Fig. 1 compares the relationship $n(\alpha)$ for the cases when $k = 0$ and $k = 1$, respectively. Also Fig. 2 compares the corresponding relation $I(\alpha)$. Having found the value of n it is easy to find from Eq. (4) the geometry of the missiles of minimum ballistic factor for given values of α in both the cases $k = 0$ and $k = 1$. Table 1 gives the comparative values of n and I for $\alpha = 0,1,2$ for the two cases $k = 0$ and $k = 1$.

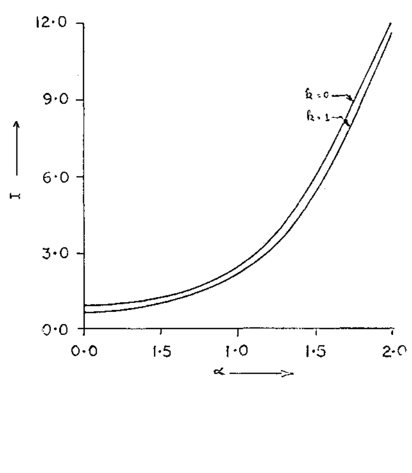

Fig. 2 Minimum quality coefficient

Table 1 Values of n and I for given values of α for the cases $k = 0$ and $k = 1$

	$k = 0$ (Newton Law)			$k = 1$ (Newton-Busemann Law)		
	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
n	0.6937	0.6780	0.6179	0.6372	0.6248	0.5822
I	1.0286	2.4354	12.1738	0.7670	2.1537	11.7767

It can be observed that with the increasing values of the friction factor α the power law exponent n decreases while the corresponding ballistic factor increases. Also the values of n and I obtained by using Newton-Busemann law are always less than those obtained by using Newton law corresponding to the same value of α.

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Flow through an Asymmetric Two-Dimensional Contraction

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A RECENT paper by Oates and Nash-Webber[1] considered the problem of an irrotational, incompressible flow in a two-dimensional channel with a contraction. The purpose of their study was to determine the nonuniformity of the axial velocity profile beyond the contraction. This is an important consideration when using such a contraction in the entrance region of a wind or water tunnel. The assumption made in Ref. 1 is that the amplitude of the contraction is small compared to the mean channel height. The problem considered in this paper is the same as above, but no restriction is imposed on the amplitude of the contraction.

The geometry of the channel is shown in Fig. 1, where $g(x)$ and $f(x)$ are continuous functions with continuous first derivatives, defined as

$$g(x) = \begin{cases} A, & x \leq -L \\ \bar{g}(x), & -L \leq x \leq \alpha L \\ a, & x \geq \alpha L \end{cases}$$

$$f(x) = \begin{cases} -A, & x \leq 0 \\ \bar{f}(x), & 0 \leq x \leq \alpha L \\ -b, & x \geq \alpha L \end{cases} \quad (1)$$

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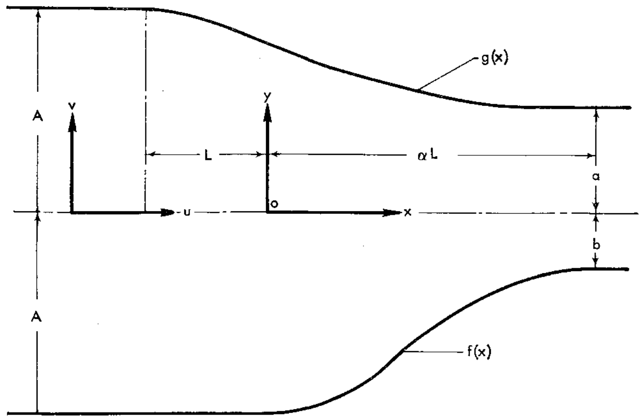

Fig. 1 Geometry of the channel.

For an irrotational, incompressible flow the velocity can be defined in terms of a potential $\phi(x,y)$, which satisfies the two-dimensional Laplace equation.

The boundary conditions are that the normal component of velocity on the walls of the channel must vanish, which leads to

$$
\begin{aligned}
(\partial\phi/\partial x)f'(x) - \partial\phi/\partial y &= 0 \text{ on } y = f(x) \\
(\partial\phi/\partial x)g'(x) - \partial\phi/\partial y &= 0 \text{ on } y = g(x)
\end{aligned}
\tag{2}
$$

The velocity potential for this type of flow is found to be

$$
\phi(x,y) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{-\infty}^{\infty} A(k) \cosh ky e^{ikx} dk + \left(\frac{1}{2\pi}\right)^{1/2} \int_{-\infty}^{\infty} B(k) \sinh ky e^{ikx} dk + U_o x \tag{3}
$$

where the uniform flow at infinity, $U_o x$, has been added to maintain irrotational flow in a simply connected region bounded by fixed walls.[2]

Equation (2) applied to Eq. (3) results in the value of $A(k)$ and $B(k)$.

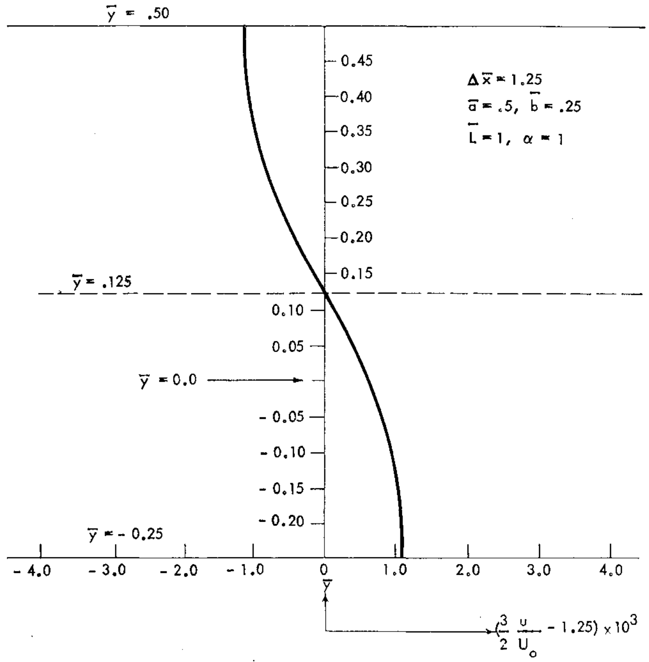

Fig. 2 Deviation of velocity from uniform flow.

where

$$
F(k) = \left(\frac{1}{2\pi}\right)^{1/2} \int_0^{\alpha L} \bar{f}'(x) e^{-ikx} dx
$$

$$
G(k) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{-L}^{\alpha L} \bar{g}'(x) e^{-ikx} dx
$$

As an example, consider the side walls to be given by the equations

$$
\bar{g}(x) = -(A - a)\sin^2[\pi(x + L)/2(\alpha + 1)L] + A
$$

$$
\bar{f}(x) = (A - b)\sin^2[\pi x/2\alpha L] - A
$$

Then

$$
\bar{u} = \frac{1}{(\bar{a} + \bar{b})}\Bigg\{ 2 - (\bar{a} + \bar{b}) + (1 - \bar{b}) \sum_{n=1}^{\infty} \left[\frac{(-1)^n \cos \dfrac{n\pi(\bar{a} - \bar{y})}{(\bar{a} + \bar{b})} \exp\left(-n \dfrac{\pi \bar{L} \Delta\bar{x}}{\bar{a} + \bar{b}}\right)}{\left[1 + \dfrac{n^2\alpha^2\bar{L}^2}{(\bar{a} + \bar{b})^2}\right]} \left(1 + \exp\left[-n\frac{\pi \bar{L}\alpha}{\bar{a} + \bar{b}}\right]\right)\right] + (1 - \bar{a}) \sum_{n=1}^{\infty} \left[\frac{(-1)^n \cos \dfrac{n\pi(\bar{b} + \bar{y})}{(\bar{a} + \bar{b})} \exp\left(-n \dfrac{\pi \bar{L} \Delta\bar{x}}{\bar{a} + \bar{b}}\right)}{\left[1 + \dfrac{n^2\bar{L}^2(\alpha + 1)^2}{(\bar{a} + \bar{b})^2}\right]} \left(1 + \exp\left[-n\frac{\pi \bar{L}(\alpha + 1)}{\bar{a} + \bar{b}}\right]\right)\right]\Bigg\}
$$

Oates et al. were interested in determining the downstream distance from the contraction where the flow was uniform. In the downstream region, $x \geq \alpha L$, the velocity components are

$$
v = \frac{\partial\phi}{\partial x} = +\frac{U_o}{(2\pi)^{1/2}} \times \int_{-\infty}^{\infty} \frac{e^{ikx}[F \sinh k(a - y) + G \sinh k(b + y)]}{\sinh k(a + b)} dk
$$

$$
u = \frac{\partial\phi}{\partial x} = -\frac{U_o i}{(2\pi)^{1/2}} \times \int_{-\infty}^{\infty} \frac{[F \cosh k(a - y) - G \cosh k(b + y)]}{\sinh k(a + b)} e^{ikx} + U_o
$$

where the barred quantities are dimensionless variables defined as follows:

$$
\bar{u} = 2u/U_o,\ \bar{L} = L/A,\ \bar{b} = b/A,\ \bar{a} = a/A
$$

$$
\bar{y} = y/A,\ \bar{x} = x/L,\ \Delta\bar{x} = \bar{x} - \alpha
$$

Typical results for $\bar{a} = 0.5$, $\bar{b} = 0.25$, $\bar{L} = 1.0$, $\alpha = 1.0$, and $\Delta\bar{x} = 1.25$ are shown in Fig. 2. The maximum deviation from uniform flow is less than 0.1%. If a greater deviation from uniform flow is allowed, then $\Delta\bar{x}$ will be less than 1.25.

As an alternative problem, one can consider $\Delta\bar{x}$ fixed and determine the required channel dimensions for uniform flow. In this case the first-order terms of the series can be examined for a first approximation. For the flow to be uniform, it follows that

$$\frac{(1-\bar{b})\left(1+\exp\left[-\dfrac{\pi\bar{L}\alpha}{\bar{a}+\bar{b}}\right]\right)}{1+\dfrac{\alpha^2\bar{L}^2}{(\bar{a}+\bar{b})^2}} = \frac{(1-\bar{a})\left(1+\exp\left[-\dfrac{\pi\bar{L}(\alpha+1)}{\bar{a}+\bar{b}}\right]\right)}{1+\dfrac{\bar{L}^2(\alpha+1)^2}{(\bar{a}+\bar{b})^2}} \tag{4}$$

which is an equation relating the dimensions of the channel. Equation (4) is equivalent to the "rule of thumb" in Ref. 1 and for the mean channel height large compared to the amplitude of the sidewall contraction, Eq. (4) reduces to the rule of thumb in Ref. 1.

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Characteristics of a Magnetic Annular Arc Operating Continuously at Atmospheric Pressure

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Magnetic Annular Arc (MAARC) devices have been operated continuously in the millibar-kilowatt range[1] and have been pulsed in the atmosphere-multimegawatt range.[2,3] The AEDC MAARC, a modified version of a device designed for low-pressure operation,[4] was operated on nitrogen exhausting to the atmosphere to determine performance trends. The electrical and thermal characteristics of this direct-current, one-half megawatt MAARC are discussed.

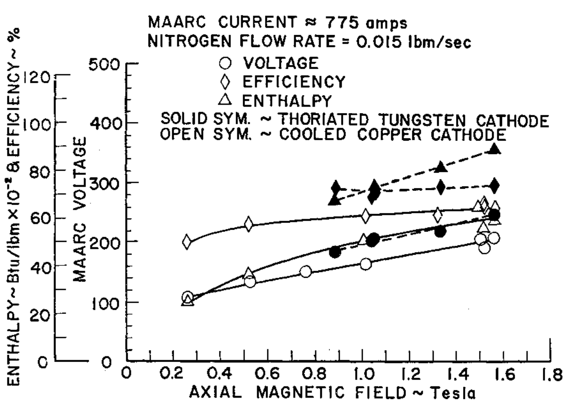

Fig. 1 Experimental data showing the effect of thoriated tungsten cathode and copper cathode on performance as the axial magnetic field is increased.

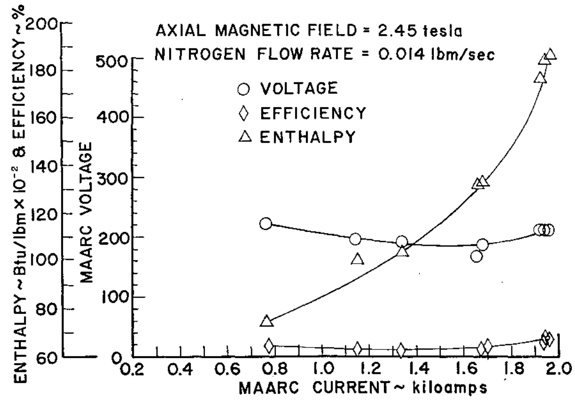

Fig. 2 The variation with current of the voltage, enthalpy, and thermal efficiency at a high axial magnetic field.

The MAARC consists of a constant area coaxial copper anode, 3.4 cm in diameter, and cathode, 2.5 cm in diameter. The cathode tip was hemispherical and both copper and thoriated tungsten were used. A primarily axial magnetic field up to 2.5 tesla (25,000 gauss) was applied at the cathode tip by an external magnet. The ratio of the radial magnetic field to the axial magnetic field in the discharge was approximately 0.11. A high-speed movie camera was used to photograph the discharge at 10,000 frames/sec with a shutter speed of two μsec. All components of the MAARC and magnet were water cooled allowing continuous operation at arc currents from 700 to 2000 amp.

Figure 1 shows the variation of enthalpy, voltage, and efficiency with increasing magnetic field for both the copper and tungsten cathodes. Operation with a tungsten cathode invariably gives a higher voltage and efficiency, thus a higher enthalpy. The cathode heat loss using tungsten is approximately one-half of that for copper. This may be attributed to the thermal emission of the tungsten tending to cool the cathode. The over-all efficiency of the device increases with magnetic field due to the decrease of the fraction of the power lost to the cathode and anode. The voltage, for both materials, increases linearly with increasing magnetic field agreeing with earlier experimental results.[1-3]

The MAARC has a drooping V-I characteristic except at high current and high magnetic fields where the slope becomes positive. This voltage characteristic, as well as the variation of enthalpy and efficiency with current, is shown in Fig. 2. The anode power loss increases linearly with increasing current, but the cathode loss is independent of current. The over-all efficiency of the device decreases slightly and then increases as the current increases. The enthalpy changes rapidly with increasing current and the maximum enthalpy measured (based on a heat balance) during the

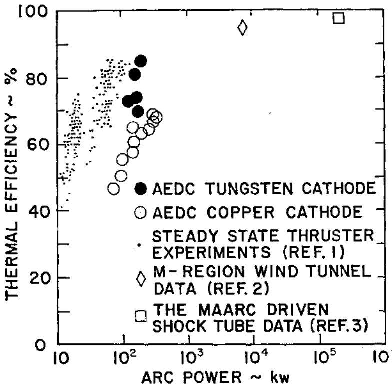

Fig. 3 Comparison of the thermal efficiency of the AEDC steady-state atmospheric MAARC with the data from the low-pressure and pulsed experiments.

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